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Analytic non-supersymmetric background dual of a confining gauge theory and the corresponding plane wave theory of hadrons

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Abstract: We find a regular analytic 1st order deformation of the Klebanov-Strassler background. From the dual gauge theory point of view the deformation describes supersymmetry soft breaking gaugino mass terms. We calculate the difference in vacuum energies between the supersymmetric and the non-supersymmetric solutions and find that it matches the field theory prediction. We also discuss the breaking of the $\mathrm{U}(1)_R$ symmetry and the space-time dependence of the gaugino bilinears two point function. Finally, we determine the Penrose limit of the non-supersymmetric background and write down the corresponding plane wave string theory. This string describes "annulons"-heavy hadrons with mass proportional to large global charge. Surprisingly the string spectrum has two fermionic zero modes. This implies that the sector in the non-supersymmetric gauge theory which is the dual of the annulons is supersymmetric.

Keywords: D-branes, Penrose limit and pp-wave background, AdS-CFT and dS-CFT Correspondence, Supersymmetry Breaking.

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1. Introduction

The AdS/CFT correspondence [1, 2, 3] (see [4] for a review) is an explicit realization of the holography principle and describes the duality between a string theory (type-IIB on $AdS_5 \times S^5$) and a gauge theory ($\mathcal{N} = 4$ SYM in four dimensions). Since the formulation of the AdS/CFT conjecture there has been great progress in the study of theories with less supersymmetries and not necessarily conformal. There are several approaches one can use to break the $\mathcal{N} = 4$ supersymmetry down to $\mathcal{N} = 2$ or $\mathcal{N} = 1$. For instance, the SUSY is broken by placing D-branes at an orbifold fixed point. One might also construct the $\mathcal{N} = 2^*$ theory by giving non-zero masses to various fields in the gauge theory. Few years ago two important examples of supergravity duals of $\mathcal{N} = 1$ gauge theories have been provided by [5] and [6] (see [7] and [8] for recent reviews). The Maldacena-Nunez (MN) background consists of NS5-branes wrapped on an S^2 and based on the solution of [9]. The supergravity dual of Klebanov-Strassler (KS) involves D5 branes wrapped around a shrinking S^2. The metric has a standard D3-form form with the 6d deformed conifold being the transversal part of the 10d space.

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Non-supersymmetric deformations of the MN background have been studied by number of authors. In [10] the supersymmetry was broken completely by giving masses for some of the scalar fields. The explicit solution was constructed in seven-dimensional gauged supergravity at leading order in the deformation and then up-lifted to ten dimensions. It was argued that the deformed non-supersymmetric background is guaranteed to be stable, since the original dual gauge theory had a mass gap. On other hand, the authors of [11] used the solution of [12] to study the supersymmetry breaking by the inclusion of a gaugino mass term and a condensate. Evidently, the global symmetry remains unbroken under this deformation.

Our main goal is to find a non-singular, non-supersymmetric deformation of the KS solution, which preserves the global symmetries of the original background and to study the Penrose limit of the new solution. The problem has been already attacked by different authors. In [13] the second order equations of motion following from the one dimensional effective action were solved perturbatively in the deep IR and UV regions. However, the numerical interpolation between the IR and UV regions does not lead to a desired relation between the corresponding parameters and the question of existence of the global non-singular solution remains unanswered. The authors of [14] suggested a computational technique for studying the non-supersymmetric solution. The technique is based on the modification of the first order BPS equations, so that we might continue to use a superpotential even for a non-supersymmetric solution. In short, one obtains a set of sixteen 1st equations and one zero-order constraint instead of eight standard 2nd order differential equations. Analyzing asymptotic solutions it was found that regularity of the IR and UV behavior restricts to three the number of possible deformations.

In this paper we determine and describe a regular *analytic* solution of the 1st order equations similar to those appearing in [14]. We note that these equations are significantly simplified once we properly redefine the radial coordinate. (The equations transform non-trivially under the coordinate redefinition since one has to apply the "zero-energy" constraint, which removes the "gauge freedom" of the coordinate transformation). We also demonstrate how part of the 1st order equations can be re-derived using the usual 2nd order IIB equations of motion.

Our solution preserves the global symmetry and therefore describes a deformation corresponding to the inclusion of mass terms of the two gaugino bilinears in the dual gauge theory.

Taking Penrose limits of both MN and KS solutions around geodesics located in the deep IR region results in solvable string theory models [15]. The string hamiltonians of these models describe the 3d non-relativistic motion and the excitations of heavy hadrons (called "annulons") with masses proportional to a large global symmetry charge $M_{\text{annulon}} = m_0 J$. It was further shown that the $\mathcal{N} = 1$ supersymmetry of the original theories implies that the world-sheet spinors have two zero-frequency modes providing a 4-dimensional Hilbert sub-space of degenerate states (two bosonic and two fermionic).

We construct a Penrose limit (see [16]–[21]) of our non-supersymmetric KS background and obtain a pp-wave metric and a complex 3-form which are very similar to the PL limit [15] of the supersymmetric solution. We also quantize the light-cone string hamiltonian and determine the masses of the bosonic and fermionic modes. These masses, though different from the supersymmetric case, still obey the relation that the sum of the mass squared is the same for bosonic and fermionic modes. Again the string describes kinematics and excitations of the annulons. The only difference between them and those of [15] is a modification of m_0. A surprising feature of the string spectrum is that, like in the Penrose limit of the KS background, here as well, there are two fermionic zero modes. In the dual field theory this implies that even though the full theory is non-supersymmetric, the sector of states with large J charge admits supersymmetry. It is conceivable that in this limit of states of large mass the impact of the small gaugino mass deformations is washed away.

The authors of [22] used the solution of [13] to take the PL. The IR expansion of the fields given in [13] differs, however, from our solution (see later) and therefore the pp-wave background of [22] is also slightly different from the metric we have derived.

The organization of the paper is as follows. In section 2 we give a short review of the KS model. Section 3 is devoted to the non-supersymmetric deformation. We start by recalling the technique of [14] and then derive and solve a set of 1st order equations using a different choice of the radial coordinate. Since the expression of the various fields in the solution are too complicated we report here only the asymptotic behavior of the fields in the UV and in the IR. In section 4 we find the vacuum energy of the supergravity solution. We further use this result in section 5 while discussing various properties of the gauge theory dual of the non-supersymmetric background. We argue that the deformation corresponds to the mass terms of the two gaugino bilinears that in particular lift the degeneracy of the vacuum. In section 6 we investigate the Penrose limit of the non-supersymmetric background. We then describe the solution of the plane wave string in 7 in the form of the annulons. We close in the last section with conclusions and suggestions for a further research. The type-IIB equations of motion and the explicit solution for some of the field are presented in two appendices

2. The Klebanov-Strassler model and beyond

Before reviewing the main features of the KS solution it will be worth to write down the type-IIB equation of motion for a case of a constant dilaton ($e^{\Phi} = g_s$), a vanishing axion ($C_0 = 0$) and with the 10d metric and the 5-form flux having the structure of the D3-brane solution, namely:

$$ds^2 = h^{-1/2}\left(\mathbf{d}x_0^2 + \cdots + \mathbf{d}x_3^2\right) + h^{1/2} ds^2_{M_6} \tag{2.1}$$

and

$$\tilde{F}_5 = \frac{1}{g_s}(1 + \star_{10})\mathbf{d}h^{-1} \wedge \mathbf{d}x_0 \wedge \cdots \wedge \mathbf{d}x_3\,, \tag{2.2}$$

where M_6 is a 6d Ricci flat transversal space and the harmonic function h depends only on the coordinates on M_6. We will denote the Hodge dual on M_6 by $\star_6$. For a general $n \leq 6$ form on M_6 there is a simple relation between $\star_{10}$ and $\star_6$:

$$\star_{10}\omega_n = h^{-\frac{n-1}{2}} \star_6 \omega_n \wedge \mathbf{d}x_0 \wedge \mathbf{d}x_1 \wedge \mathbf{d}x_2 \wedge \mathbf{d}x_3\,. \tag{2.3}$$

With this observation in mind and assuming that the 3-forms have their legs only along M_6 we derive the 6d version of the dilaton and axion equations of motion:

$$
\begin{aligned}
0 &= -H_3 \wedge \star_6 H_3 + g_s^2 F_3 \wedge \star_6 F_3 \\
0 &= H_3 \wedge \star_6 F_3 \,.
\end{aligned}
\tag{2.4}
$$

In order to find the connection between the 3-forms and the warp function h we have to use the 5-form equation. We end up with:

$$
\mathbf{d} \star_6 \mathbf{d} h = g_s H_3 \wedge F_3 \tag{2.5}
$$

or

$$
\tilde{F}_5 = B_2 \wedge F_3 + \star_{10} \left(B_2 \wedge F_3 \right) . \tag{2.6}
$$

In what follows we will adopt the integrated version of (2.5):

$$
\mathbf{d} h = -g_s \star_6 \left(B_2 \wedge F_3 \right) . \tag{2.7}
$$

On equal footing we might replace (2.7) by

$$
\mathbf{d} h = g_s \star_6 \left(C_2 \wedge H_3 \right) . \tag{2.8}
$$

Note that the difference between the warp functions satisfying (2.7) and (2.8) is precisely the harmonic function $\tilde{h}(\tau)$ on the deformed conifold satisfying $\bar{\nabla}_2^6 \tilde{h} = 0$. This function, however, diverges at $\tau \to 0$. We can therefore use one of the integrated versions of (2.5) together with the requirement of regularity of $h(\tau)$ at $\tau = 0$.

Next we consider the 3-forms equations. Applying (2.7) and the relation between F_5 and $\tilde{F}_5$ we get:

$$
\mathbf{d}\left[h^{-1} \left(\star_6 F_3 + \frac{1}{g_s} H_3 \right) \right] = 0 \qquad \text{and} \qquad \mathbf{d}\left[h^{-1} \left(\star_6 H_3 - g_s F_3 \right) \right] = 0 \tag{2.9}
$$

In deriving this result we have used the fact that all the forms have their legs along the 6d space and therefore $(C_2 \wedge H_3 + B_2 \wedge F_3) \wedge H_3 = 0$. Finally, we re-write the metric equation of motion. Remarkably, for the metric and the forms considered in our case it is enough to verify only the trace of the Einstein equation. Calculating the Ricci scalar of the metric (2.1) we find:

$$
R = \frac{1}{2} h^{-3/2} \nabla_6^2 h = \frac{1}{2} h^{-3/2} \star_6 \mathbf{d} \star_6 \mathbf{d} h \,. \tag{2.10}
$$

Recalling the self-duality of $\tilde{F}_5$ and using again the relation between $\star_{10}$ and $\star_6$ we obtain:

$$
\mathbf{d} \star_6 \mathbf{d} h = \frac{1}{2} \left[H_3 \wedge \star_6 H_3 + g_s^2 F_3 \wedge \star_6 F_3 \right] . \tag{2.11}
$$

The equations we have written (2.4), (2.7), (2.9), (2.11) are easily solved by requiring that:

$$
\star_6 F_3 = -g_s^{-1} H_3 \qquad \text{and} \qquad \star_6 H_3 = g_s F_3 \,. \tag{2.12}
$$

In this case the complex form $G_3 \equiv F_3 + \frac{i}{g_s} H_3$ is imaginary self dual $\star_6 G_3 = i G_3$.

Note that the equation for h is a first order differential equation, even though the solution is not supersymmetric in general.

The most important example of the supersymmetric solution is the Klebanov-Strassler model [6], where the 6d manifold is the deformed conifold space. The 6d metric is given by:

$$ds_6^2 = \frac{1}{2}\epsilon^{4/3}K(\tau)\Bigg[\frac{1}{K^3(\tau)}\left(d\tau^2+(g^5)^2\right)+\cosh^2\left(\frac{\tau}{2}\right)\left((g^3)^2+(g^4)^2\right)+ + \sinh^2\left(\frac{\tau}{2}\right)\left((g^1)^2+(g^2)^2\right)\Bigg], \tag{2.13}$$

where

$$K(\tau) = 2^{-1/3}\frac{(\sinh 2\tau - 2\tau)^{1/3}}{\sinh\tau}. \tag{2.14}$$

At large τ we may use another radial coordinate defined by:

$$r \to \epsilon^{2/3}e^{\tau/3}. \tag{2.15}$$

In terms of r we have:

$$ds_6^2 \to dr^2 + r^2 d\Omega_{T_{1,1}}^2. \tag{2.16}$$

The determinant of (2.13) vanishes at $\tau = 0$ reflecting the fact the 6d metric degenerates into the metric of S^3.

The $\mathbf{M}$ fractional D5-branes wrapping the shrinking S^2 are introduced through the RR 3-form:

$$F_3 = \mathbf{M}\Big[(1-F(\tau))g^5\wedge g^3\wedge g^4 + F(\tau)g^5\wedge g^1\wedge g^2 + F'(\tau)d\tau\wedge\left(g^1\wedge g^3+g^2\wedge g^4\right)\Big], \tag{2.17}$$

together with the boundary conditions $F(0) = 0$ and $F(\infty) = \frac{1}{2}$. The former condition ensures that F_3 is proportional to the volume form of the non-collapsing S^3 at $\tau = 0$, while the later means the restoration of the $\mathrm{U}(1)_R$ symmetry in the UV, where we approach the geometry of the conifold over $T_{1,1}$ [6]. On using the duality relations (2.12) one may find the NS 3-form:

$$H_3 = \mathbf{d}B_2 = g_s\mathbf{M}\Big[f'(\tau)d\tau\wedge g^1\wedge g^2 + k'(\tau)d\tau\wedge g^3\wedge g^4 + + \frac{1}{2}\left(k(\tau)-f(\tau)\right)g^5\wedge\left(g^1\wedge g^3+g^2\wedge g^4\right)\Big], \tag{2.18}$$

where the functions $f(\tau)$,$k(\tau)$ and $F(\tau)$ satisfy:

$$f'(\tau) = (1-F(\tau))\tanh^2\left(\frac{\tau}{2}\right), \qquad k'(\tau) = F(\tau)\coth^2\left(\frac{\tau}{2}\right), \tag{2.19}$$

and

$$F'(\tau) = \frac{k(\tau)-f(\tau)}{2}. \tag{2.20}$$

This system of the first order differential equations has three dimensional space of solutions:

$$F(\tau) = \frac{1}{2} - \frac{\tau}{2\sinh\tau} + C_1\left(\cosh\tau - \frac{\tau}{\sinh\tau}\right) + \frac{C_2}{\sinh\tau}$$

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$$
\begin{aligned}
f(\tau) &= \frac{\tau\coth\tau-1}{2\sinh\tau}(\cosh\tau-1)+\\
&\quad +C_1\left(2\tau-\sinh\tau-\tanh\frac{\tau}{2}-\frac{\tau}{2\cosh^2\frac{\tau}{2}}\right)+\frac{C_2}{2\cosh^2\frac{\tau}{2}}+C_3\\
k(\tau) &= \frac{\tau\coth\tau-1}{2\sinh\tau}(\cosh\tau+1)+\\
&\quad +C_1\left(2\tau+\sinh\tau-\coth\frac{\tau}{2}+\frac{\tau}{2\sinh^2\frac{\tau}{2}}\right)-\frac{C_2}{2\sinh^2\frac{\tau}{2}}+C_3\,. \qquad (2.21)
\end{aligned}
$$

The KS solution corresponds to $C_1 = C_2 = C_3 = 0$. Using the complex structure of the deformed conifold space the complex form $G_3 = F_3 + \frac{i}{g_s}H_3$ can be identified in this case as a $(2,1)$ form [23]. Instead, for $C_1 = -1/2$ and $C_2 = C_3 = 0$ we obtain a $(0,3)$ form which breaks the supersymmetry and diverges at $\tau \to \infty$. In appendix C we prove this statement by performing an explicit calculation making use of the complex structure of the deformed conifold given in [24] (similar derivation can be done using the results of [25] and [23]).[1] For $C_2 \neq 0$ we find another $(2,1)$ form which is singular at $\tau = 0$. Finally, the solution with $C_3 \neq 0$ amounts to the gauge freedom $B_2 \to B_2 + \mathbf{d}A_1$.

Having determined the 3-forms one can integrate (2.7) to find the harmonic function $h(\tau)$:

$$
h(\tau) = \alpha\frac{2^{1/3}}{4}\int_\tau^\infty dx\frac{x\coth x-1}{\sinh^2 x}(\sinh(2x)-2x)^{1/3}\,. \qquad (2.22)
$$

The asymptotic behavior of $h(\tau)$ is:

$$
h(\tau\to 0)\to a_0 \qquad \text{and} \qquad h(\tau\to\infty)\to\alpha\frac{3}{4}2^{1/3}\tau e^{-4\tau/3} \qquad (2.23)
$$

with $\alpha \equiv 4(g_s\mathbf{M}l_s^2)^2\epsilon^{-8/3}$.

The dual field theory realized on the world-volume of the $\mathbf{N}$ physical and $\mathbf{M}$ fractional D3-branes is a 4d $\mathcal{N} = 1$ supersymmetric $\mathrm{SU}(\mathbf{N}+\mathbf{M}) \times \mathrm{SU}(\mathbf{N})$ gauge theory with a $\mathrm{SU}(2)\times\mathrm{SU}(2)$ global symmetry inherited from the conifold isometries. The gauge theory is coupled to two bi-fundamental chiral multiplets A and B, which transform as a doublet of one of the SU(2)'s each and are inert under the second SU(2). This theory is believed to exhibit a cascade of Seiberg dualities reducing in the deep IR to pure $\mathrm{SU}(\mathbf{M})$. On the supergravity side $\mathbf{M}$ is fixed by the charge of the RR 3-form, while $\mathbf{N}$ is encoded in the UV behavior of the 5-form:

$$
\mathcal{F}_5 \sim \mathbf{N}_{\text{eff}}\mathrm{Vol}(T_{1,1}), \qquad \text{where} \qquad \mathbf{N}_{\text{eff}} = \mathbf{N} + \frac{3}{2\pi}g_s\mathbf{M}^2\ln\frac{r}{r_0}\,. \qquad (2.24)
$$

The sum of the gauging couplings is constant and the logarithmic running of the difference is determined by the NS 2-form:

$$
\frac{1}{g_1^2}+\frac{1}{g_2^2}\sim e^{-\Phi} \qquad \frac{1}{g_1^2}-\frac{1}{g_2^2}\sim e^{-\Phi}\left[\int_{S^2}B_2-\frac{1}{2}\right]. \qquad (2.25)
$$

[1]Note that the deformed conifold is a non-compact space and therefore there is no obstacle for constructing a closed $(0,3)$ form, which is regular everywhere except at infinity.

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Similarly to pure SU($\mathbf{M}$) the theory confines. This is evident by virtue of the fact that the warp factor approaches a constant value $h_0 \sim a_0$ at $\tau \to 0$ and therefore the tension of the confining strings does not diverge. This conclusion is valid only for a non-zero value of the deformation parameter ϵ, since $a_0 \sim \epsilon^{-8/3}$. Note also that for $\epsilon \neq 0$ the U$(1)_R$ conifold symmetry is broken down to $\mathbf{Z}_2$. This is the symmetry preserved by the gaugino bilinear $\mathrm{Tr}\,\lambda\lambda(x)$. In the supergravity dual this gauge theory operator is associated with the form $C_2 = C_2^{RR} + iB_2^{NS}$ [26]. Subtracting the asymptotic value of $G_3 = \mathbf{d}C_2$ we find at $\tau \to \infty$:

$$\Delta G_3 \approx \frac{1}{2}\mathbf{M}\tau e^{-\tau}\omega_3\,, \qquad \omega_3 = g^5 \wedge \left[\left(g^3 \wedge g^4 - g^1 \wedge g^2\right) + ig_s\left(g^1 \wedge g^3 + g^2 \wedge g^4\right)\right], \tag{2.26}$$

where we write only the polarization along $T_{1,1}$. Similarly:

$$\Delta C_2 \approx -\frac{1}{2}\mathbf{M}\tau e^{-\tau}\omega_2\,, \qquad \omega_2 = \left[\left(g^1 \wedge g^3 + g^2 \wedge g^4\right) + ig_s\left(g^1 \wedge g^2 - g^3 \wedge g^4\right)\right] \tag{2.27}$$

and we see that ΔC_2 transforms under U$(1)_R$ by the same phase as $\mathrm{Tr}(\lambda\lambda)$. Moreover, ΔG_3 has an asymptotic behavior we would expect from a scalar operator of dimension 3 and a non-zero VEV, namely:

$$\Delta G_3 = \frac{1}{2}\mathbf{M}\frac{m^3}{r^3}\ln\frac{r^3}{m^3}\omega_3\,, \tag{2.28}$$

where the deformation parameter is related to the 4d mass scale through $m \sim \epsilon^{2/3}$.

Finally, we will recall the identification of supergravity fields with gauge theory operators. In order to find this correspondence one writes the most general SU(2) $\times$ SU(2) invariant background ansatz, which includes the supersymmetric KS solution:

$$\begin{aligned}
ds^2 &= 2^{1/2}3^{3/4}\Big[e^{-5q(\tau)+2Y(\tau)}\left(dx_\mu dx^\mu\right) + +\frac{1}{9}e^{3q(\tau)-8p(\tau)}\left(d\tau^2 + g_5^2\right) + \\
&\qquad + \frac{1}{6}e^{3q(\tau)+2p(\tau)+y(\tau)}\left(g_1^2 + g_2^2\right) + \frac{1}{6}e^{3q(\tau)+2p(\tau)-y(\tau)}\left(g_3^2 + g_4^2\right)\Big] \\
B_2 &= -\left(\tilde{f}(\tau)g_1 \wedge g_2 + \tilde{g}(\tau)g_3 \wedge g_4\right), \qquad \Phi = \Phi(\tau)\,, \\
F_3 &= 2Pg_5 \wedge g_3 \wedge g_4 + \mathbf{d}\left[\tilde{F}(\tau)\left(g_1 \wedge g_3 + g_2 \wedge g_4\right)\right], \\
\tilde{F}_5 &= \mathcal{F}_5 + \star_{10}\mathcal{F}_5\,, \qquad \mathcal{F}_5 = -\tilde{L}(\tau)g_1 \wedge g_2 \wedge g_3 \wedge g_4 \wedge g_5\,, \\
\tilde{L}(\tau) &= Q + \tilde{f}(\tau)(2P - \tilde{F}(\tau)) + \tilde{k}(\tau)\tilde{F}(\tau)\,.
\end{aligned} \tag{2.29}$$

This general ansatz includes both the conformal solution with a singular geometry ($y = \tilde{f} - \tilde{k} = 0$) and the non-conformal case with regular deformed conifold ($y, \tilde{f} - \tilde{k} \neq 0$). Here $\tilde{f}, \tilde{k}$ and $\tilde{F}$ are the rescaled KS functions:

$$\tilde{f} = -2Pg_s f\,, \qquad \tilde{k} = -2Pg_s k\,, \qquad \tilde{F} = 2PF \tag{2.30}$$

and the constants Q and P are related to the number of physical and fractional branes respectively: $P = \frac{1}{4}\mathbf{M}l_s^2$ and Q is proportional to $\mathbf{N}$, but for $P \neq 0$, it can be re-absorbed in the redefinition of $\tilde{f}$ and $\tilde{k}$.[2] Note that for the given structure of the 3-form F_3 the integral $\int_{S_3} F_3$ does not depend on $\tilde{F}(\tau)$. Moreover, the NS-NS 3-form has the same structure as in the KS solution as dictated by the equation for a vanishing axion $H_3 \wedge F_3 = 0$.

[2] In what follows we will use the $Q = 0$ convention

In the next section we will also use another parameterization of the 10d metric:

$$ds^2 = h^{-1/2}\left(dx_\mu dx^\mu\right) + h^{1/2}\left(e^{m-\frac{n}{2}}\left(d\tau^2 + g_5^2\right) + +e^{\frac{n}{2}+y}\left(g_1^2+g_2^2\right) + e^{\frac{n}{2}-y}\left(g_3^2+g_4^2\right)\right). \tag{2.31}$$

In particular in the supersymmetric case we have:

$$e^{m_0(\tau)} = \frac{\epsilon^{8/3}}{24}K(\tau)^{-1}\sinh\tau\,, \qquad e^{n_0(\tau)} = \frac{\epsilon^{8/3}}{16}K^2(\tau)\sinh^2\tau\,, \qquad e^{y_0(\tau)} = \tanh\left(\frac{\tau}{2}\right). \tag{2.32}$$

The two parameterizations are connected by:

$$e^{10q(\tau)-4Y(\tau)} = 3^{3/2}2h(\tau)\,, \qquad e^{4p(\tau)+4Y(\tau)-4q(\tau)} = \frac{1}{3}e^n(\tau)\,, \qquad e^{-10p(\tau)} = \frac{3}{2}e^{m(\tau)-n(\tau)}\,. \tag{2.33}$$

Integration of the type-IIB lagrangian over the angular and the world-volume coordinates yields a 1d effective action [27, 28, 24, 29]:

$$S \sim \int d\tau\left(-\frac{1}{2}G_{ij}\dot\phi^i\dot\phi^j - V(\phi)\right), \tag{2.34}$$

where[3]

$$\begin{aligned} G_{ij}\dot\phi^i\dot\phi^j &= e^{4p-4q+4Y}\Bigg(-18\dot Y^2 + 45\dot q^2 + 30\dot p^2 + \frac{3}{2}\dot y^2 + \frac{3}{4}\dot\Phi^2 + \\ &\qquad + 3e^{-\Phi-6q-4p}\left(\frac{\sqrt{3}}{2}e^{-2y}\dot{\tilde f}^2 + \frac{\sqrt{3}}{2}e^{2y}\dot{\tilde k}^2\right) + 3\sqrt{3}e^{\Phi-6q-4p}\dot{\tilde F}^2\Bigg) \\ V(\phi) &= e^{4Y}\Bigg(\frac{1}{3}e^{-16p-4q} - 2e^{-6p-4q}\cosh y + \frac{3}{4}e^{4p-4q}\sinh^2 y + \\ &\quad + \frac{3\sqrt{3}}{4}e^{\Phi-10q+2y}(2P-\tilde F)^2 + \frac{3\sqrt{3}}{4}e^{\Phi-10q-2y}\tilde F^2 + \\ &\quad + \frac{3\sqrt{3}}{8}e^{-\Phi-10q}(\tilde k-\tilde f)^2 + \frac{9}{2}e^{-4p-16q}\tilde L^2\Bigg). \end{aligned} \tag{2.35}$$

There is also a "zero-energy" constraint:

$$\frac{1}{2}G_{ij}\dot\phi^i\dot\phi^j - V(\phi) = 0 \tag{2.36}$$

This lagrangian admits a superpotential

$$V = \frac{1}{8}G^{ij}\frac{\partial W}{\partial\phi^i}\frac{\partial W}{\partial\phi^j}$$

for

$$W = -3e^{4Y+4p-4q}\cosh y - 2e^{4Y-6p-4q} - 3\sqrt{3}e^{4Y-10q}\tilde L \tag{2.37}$$

and for supersymmeric solutions the second order equations of motion can be reduced to the first order ones:

$$\frac{d\phi^i}{d\tau} = \frac{1}{2}G^{ij}\frac{\partial W}{\partial\phi^j}\,. \tag{2.38}$$

[3]Here we adopt the conventions of [8].

The potential appearing in the action has an $\mathcal{N}=1$ critical point corresponding to the conformal background $AdS_5 \times T_{1,1}$ generated by physical D3-branes in absence of fractional branes ($P=0$). Expanding the potential around the critical point and using the mass/dimension formula $\Delta = 2+\sqrt{4+m^2}$ one obtains the dimensions of the fields, which now can be identified with various gauge theory operators [30, 29]. Here we list two of them:

$$
\begin{aligned}
y &\to \mathrm{Tr}\left(W_{(1)}^2 - W_{(2)}^2\right) \qquad \Delta = 3\,, \\
\xi_2 \sim -F + \frac{k-f}{2} &\to \mathrm{Tr}\left(W_{(1)}^2 + W_{(2)}^2\right) \qquad \Delta = 3\,.
\end{aligned}
\tag{2.39}
$$

There are also two massless fields. $s = f+k$ is associated with a marginal direction in the CFT and the corresponding operator is $\mathrm{Tr}(F_{(1)}^2 - F_{(2)}^2)$. Similarly, the dilaton Φ corresponds to $\mathrm{Tr}(F_{(1)}^2 + F_{(2)}^2)$.

In this paper we will focus on the non-supersymmetric deformation of the KS background by introducing mass terms of the gaugino bilinears associated with both ξ_2 and y. The former field is related to the SUGRA 3-forms and the later is responsible for a deformation of the 6d metric. The expected UV behavior of the fields in the background deformed by the masses is $g(\tau)e^{-\tau/3}$, where $g(\tau)$ is a polynomial in τ.

To conclude this section let us add a remark supporting the correspondence (2.39). As we have already discussed inspecting the UV behavior of the 3-form G_3 one can identify it with a gaugino bilinear in the gauge theory. This observation is related to the second line of (2.39). To justify the first line in a similar way let us expand the 10d metric (2.31) at $\tau \to \infty$. Keeping only the parts including $g_1, \ldots, g_4$ (note that g_5 is invariant under $\mathrm{U}(1)_R$) and omitting the overall factor we get:

$$
g_1^2 + g_2^2 + g_3^2 + g_4^2 + 2e^{-\tau}\left(g_1^2 - g_3^2 + g_2^2 - g_4^2\right)\,,
\tag{2.40}
$$

where we used the expansion $e^{y_0} \sim 1 - 2e^{-\tau} + \ldots$. Thus, much like the $(0,3)$ form case, the sub-leading term of the 6d metric transforms under $\mathrm{U}(1)_R$ similarly to ξ_2 and hence breaks the $\mathrm{U}(1)_R$ symmetry. Moreover it has the dimension of the supergravity dual of the gaugino bilinear matching the relation in (2.39).

3. Non-supersymmetric extension of KS

We start this section with a brief review of the method proposed by [14] (see also [31]–[36]) to study first order non-supersymmetric deformations of the KS background still making use of the superpotential. We expand the fields around a given supersymmetric solution derived from the superpotential:

$$
\phi^i = \phi_0^i + \delta \cdot \bar{\phi}^i + O(\delta^2)\,.
\tag{3.1}
$$

Define new functions:

$$
\xi_i = G_{ij}(\phi_0)\left(\frac{d\bar{\phi}^j}{d\tau} - M_k^j(\phi_0)\bar{\phi}^k\right) \qquad \text{where} \qquad M_k^j = \frac{1}{2}\frac{\partial}{\partial\phi^k}\left(G^{jl}\frac{\partial W}{d\phi^l}\right).
\tag{3.2}
$$

Now one might represent the linearized equations of motion as a "double" set of first order equations:

$$\begin{aligned}\frac{d\xi_i}{d\tau} + \xi_j M_i^j(\phi_0) &= 0 \\ \frac{d\bar{\phi}^i}{d\tau} - M_j^i(\phi_0)\bar{\phi}^j &= G^{ik}(\phi_0)\xi_k\,. \end{aligned} \tag{3.3}$$

The second line follows trivially from the definition of ξ_i, while the first one is demonstrated by substituting the expansion (3.1) into the equations of motion (we refer the reader to [14] for the proof). Finally, the zero-energy condition can be rephrased as:

$$\xi_k \frac{d\bar{\phi}^k}{d\tau} = 0\,. \tag{3.4}$$

An important remark is in order. One can use various definitions for the radial coordinate in the 1d effective action. This ambiguity is removed by applying the zero-energy constraint. The explicit form of the 1st order equations (3.3) is highly dependent on the radial coordinate choice. In our paper we will fix this "gauge freedom" by requiring that even in the deformed solution the $G_{\tau\tau}$ and G_{55} entries of the metric will remain equal exactly as in the supersymmetric case. We will see that with this choice the set of the equations (3.3) possesses an analytic solution. On the contrary the radial coordinate ($\tau_\star$) of [14] is related to our coordinate (τ) via $d\tau_\star = e^{4\bar{p}-4\bar{q}}d\tau$. Note, however that since both $\bar{p}(\tau)$ and $\bar{q}(\tau)$ are expected to vanish at $\tau \to 0$ and $\tau \to \infty$ the deep UV and IR expansions of the fields have to be the same in terms of τ and $\tau_\star$.

Let us first write the equations of motion for ξ_i's:[4]

$$\begin{aligned}
\dot{\xi}_Y &= 0 \\
\dot{\xi}_q &= 2\sqrt{3}e^{-4p_0-6q_0}\tilde{L}_0\left(\xi_Y+\xi_q\right) \\
\dot{\xi}_p &= \frac{4}{3}\sqrt{3}e^{-4p_0-6q_0}\tilde{L}_0\left(\xi_Y+\xi_q\right) + e^{-10p_0}\left(\frac{20}{9}\xi_Y + \frac{8}{9}\xi_q + 2\xi_p\right) \\
\dot{\xi}_y &= -\left(\frac{1}{3}\xi_Y + \frac{2}{15}\xi_q - \frac{1}{5}\xi_p\right)\sinh y_0 + \xi_y\cosh y_0 + 2e^{2y_0}(2P-\tilde{F}_0)\xi_{\tilde{f}} - 2e^{-2y_0}\tilde{F}_0\xi_{\tilde{k}} \\
\dot{\xi}_{\tilde{f}+\tilde{k}} &= -\frac{\sqrt{3}}{3}2Pe^{-4p_0-6q_0}\left(\xi_Y+\xi_q\right) \\
\dot{\xi}_{\tilde{f}-\tilde{k}} &= -\xi_{\tilde{F}} - \frac{2\sqrt{3}}{3}(P-\tilde{F}_0)e^{-4p_0-6q_0}\left(\xi_Y+\xi_q\right) \\
\dot{\xi}_{\tilde{F}} &= -\left(\cosh(2y_0)\xi_{\tilde{f}-\tilde{k}} + \sinh(2y_0)\xi_{\tilde{f}+\tilde{k}}\right) - \frac{\sqrt{3}}{3}(\tilde{k}_0-\tilde{f}_0)e^{-4p_0-6q_0}\left(\xi_Y+\xi_q\right) \\
\dot{\xi}_\Phi &= \left(e^{2y_0}(2P-\tilde{F}_0)\xi_{\tilde{f}} + e^{-2y_0}\tilde{F}_0\xi_{\tilde{k}}\right) - \frac{\tilde{k}_0-\tilde{f}_0}{2}\xi_{\tilde{F}}\,,
\end{aligned} \tag{3.5}$$

where $\xi_{\tilde{f}\pm\tilde{k}} = \xi_{\tilde{f}} \pm \xi_{\tilde{k}}$. Through out this paper we will be interested in a solution satisfying:

$$\xi_Y = \xi_p = \xi_q = 0\,. \tag{3.6}$$

[4] We will set $g_s = 1$ through out this section

Under this assumption we have $\xi_{\tilde{f}+\tilde{k}} = X$ for constant X and from the equations for $\xi_{\tilde{f}-\tilde{k}}$ and $\xi_{\tilde{F}}$ we obtain:

$$\frac{d^2\xi_{\tilde{f}-\tilde{k}}}{d\tau^2} = \frac{1}{2}\left(e^{2y_0}\left(\xi_{\tilde{f}-\tilde{k}} + X\right) + e^{-2y_0}\left(\xi_{\tilde{f}-\tilde{k}} - X\right)\right). \tag{3.7}$$

This equation has a two dimensional space of solutions. However, solving for ξ_y, plugging the result into the zero-energy constraint $\xi_i\dot{\phi}_0^i = 0$ and requiring also regularity at $\tau \to 0$ (otherwise we might obtain a singular solution for the fields) we pick up a unique simple solution $\xi_{\tilde{f}-\tilde{k}}(\tau) = X\cosh\tau$. To summarize we have the following result for ξ_i's:

$$\begin{aligned}\xi_{\tilde{f}} &= \frac{1}{2}X(\cosh\tau + 1)\,, \qquad \xi_{\tilde{k}} = \frac{1}{2}X(-\cosh\tau + 1)\,, \qquad \xi_{\tilde{F}} = -X\sinh\tau\,,\\ \xi_y &= 2PX(\tau\cosh\tau - \sinh\tau) \qquad \text{and} \qquad \dot{\xi}_\Phi = 0\,,\end{aligned} \tag{3.8}$$

where the last result can be easily verified by a straightforward calculation. Having determined the explicit form of ξ_i's we can consider the equations for the fields $\bar{\phi}^i$'s. First we write the equation for $\bar{\Phi}(\tau)$:

$$\dot{\bar{\Phi}} = \frac{4}{3}e^{4q_0-4p_0-4Y_0}\xi_\Phi\,. \tag{3.9}$$

Since ξ_Φ is constant the unique solution which is regular at $\tau \to 0$ corresponds to $\xi_\Phi = 0$ and therefore $\dot{\bar{\Phi}} = 0$. For $\bar{y}$ we get:

$$\dot{\bar{y}} + \cosh(y_0)\bar{y} = \frac{2}{3}e^{4q_0-4p_0-4Y_0}\xi_y\,. \tag{3.10}$$

Using the result for ξ_y and substituting the expressions for $q_0(\tau)$, $p_0(\tau)$ and $Y_0(\tau)$ we may solve for $\bar{y}(\tau)$:

$$\bar{y}(\tau) = 32\epsilon^{-8/3}2^{2/3}\frac{2PX}{\sinh\tau}\int_0^\tau \frac{(x\coth x - 1)\sinh^2 x}{(\sinh(2x) - 2x)^{2/3}}dx\,, \tag{3.11}$$

where we fixed the integration constant by requiring regularity at $\tau \to 0$. For our purposes we need an asymptotic behavior of $\bar{y}(\tau)$. At $\tau \to \infty$ we have:

$$\bar{y} \approx \mu\left(\tau - \frac{5}{2}\right)e^{-\tau/3} + Ve^{-\tau} + \cdots\,, \tag{3.12}$$

where $\mu = 48\epsilon^{-8/3}2^{1/3}2Pg_sX$ and V is a numerical constant proportional to μ. Note that μ is a dimensionless parameter (the dimensions of ϵ, P and X are $-\frac{3}{2}$, -2 and -2 respectively). In the IR we find:

$$\bar{y}(\tau) = \frac{3^{2/3}}{27}\mu\tau^2 + O(\tau^4)\,. \tag{3.13}$$

The equation for $\bar{p}(\tau)$ is given by:

$$\dot{\bar{p}} + \frac{1}{5}\sinh(y_0)\bar{y} + 2e^{-10p_0}\bar{p} = \frac{1}{30}e^{4q_0-4p_0-4Y_0}\xi_p\,. \tag{3.14}$$

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Using the result for $\bar{y}(\tau)$ and the fact that $\xi_p = 0$ we may find the solution for $\bar{p}(\tau)$. Again we require regularity at $\tau \to 0$. We get:

$$\bar{p}(\tau) = \frac{1}{5}\frac{1}{\beta(\tau)}\int_0^\tau \beta(\tau')\frac{\bar{y}(\tau')}{\sinh\tau'}d\tau' \qquad \text{where} \qquad \beta(\tau) \equiv e^{2\int_{\tau_0}^{\tau} e^{-10p_0(x)}dx}\,. \tag{3.15}$$

The solution has the following asymptotic behavior:

$$\bar{p}(\tau) \approx \frac{3}{5}\mu\,(\tau - 4)\,e^{-4\tau/3} \qquad \text{at } \tau \to \infty \tag{3.16}$$

and

$$\bar{p}(\tau) = \frac{3^{2/3}}{675}\mu\tau^2 + O(\tau^4) \qquad \text{at } \tau \to 0\,. \tag{3.17}$$

Next we consider the equation for $(\bar{Y} - \bar{q})$:

$$(\dot{\bar{Y}} - \dot{\bar{q}}) - \frac{1}{5}\sinh(y_0)\bar{y} + \frac{4}{3}e^{-10p_0}\bar{p} = -e^{4q_0-4p_0-4Y_0}\left(\frac{1}{18}\xi_Y + \frac{1}{45}\xi_q\right)\,. \tag{3.18}$$

In this case we fix the integration constant requiring that the function vanishes at infinity. The result is:

$$\bar{Y}(\tau) - \bar{q}(\tau) = \int_\tau^\infty \left(\frac{1}{5}\frac{\bar{y}(x)}{\sinh x} + \frac{4}{3}e^{-10p_0(x)}\bar{p}(x)\right)dx\,, \tag{3.19}$$

so that:

$$\bar{Y}(\tau) - \bar{q}(\tau) \approx \frac{9}{10}\mu\left(\tau - \frac{11}{4}\right)e^{-4\tau/3} \qquad \text{at } \tau \to \infty \tag{3.20}$$

and

$$\bar{Y} - \bar{q} = C_Y^0 - 3^{2/3}\frac{7}{1350}\mu\tau^2 + O(\tau^4) \qquad \text{at } \tau \to 0\,, \tag{3.21}$$

where C_Y^0 is a numerical constant proportional to μ. Now we are in a position to write down the equations for the 3-form fields. Using the expressions for $\xi_{\tilde{f}\pm\tilde{k}}$ and $\xi_{\tilde{F}}$, passing from $\tilde{f}$, $\tilde{k}$ and $\tilde{F}$ to f, k and F we obtain and recalling that $\bar{\Phi} = 0$:

$$\begin{aligned}
\dot{\bar{f}} + e^{2y_0(\tau)}\bar{F} - 2\dot{f}_0(\tau)\bar{y} &= -\frac{2X}{2P}h_0(\tau)(\cosh\tau - 1)\\
\dot{\bar{k}} - e^{-2y_0(\tau)}\bar{F} + 2\dot{k}_0(\tau)\bar{y} &= \frac{2X}{2P}h_0(\tau)(\cosh\tau + 1)\\
\dot{\bar{F}} - \frac{1}{2}(\bar{k} - \bar{f}) &= -\frac{2X}{2P}h_0(\tau)\sinh\tau\,.
\end{aligned} \tag{3.22}$$

Before discussing the explicit solution of this system it is worth to re-derive these equations using the 2nd order type-IIB equations of motion. In the most general ansatz preserving the global symmetry the 5-form $\tilde{F}_5$ is given by

$$\tilde{F}_5 = \frac{1}{g_s}(1 + \star_{10})\mathbf{d}\varphi \wedge \mathbf{d}x_0 \wedge \cdots \wedge \mathbf{d}x_3, \tag{3.23}$$

where $\varphi = \varphi(\tau)$. Supersymmetry requires $\varphi = h^{-1}$ (see [37] and [38]) , but it does not necessarily hold in a non-supersymmetric case. In what follows we will demonstrate how assuming that $\dot{\Phi} = 0$ and $\varphi = h^{-1}$ one may reproduce (3.22) from the usual 2nd order 3-forms equations of motion. Indeed, under these assumptions the type-IIB 3-forms equations reduce to (2.9). Let us expand (2.9) around the supersymmetric KS solution. Note that the expansion includes also $\star_6$ due to the deformation of the 6d space. We will denote the modified Hodge star operation by $\star_6 = \star_6^{(0)} + \bar{\star}_6$, where $\star_6^{(0)}$ corresponds to the supersymmetric configuration. After some algebra the linearized RR 3-form equation reduces to:

$$\mathbf{d} Z_3 = 0\,, \tag{3.24}$$

where

$$\begin{aligned} Z_3 &= h_0^{-1}\left(\bar{H}_3 + \star_6^{(0)}\bar{F}_3 + \bar{\star}_6 F_3^{(0)}\right) \\ &= h_0^{-1}\Big[\left(\dot{\bar{f}} + e^{2y_0}\bar{F} - 2\dot{f}_0\bar{y}\right) d\tau\wedge g^1\wedge g^2 + \left(\dot{\bar{k}} - e^{-2y_0}\bar{F} + 2\dot{k}_0\bar{y}\right) d\tau\wedge g^3\wedge g^4 + \\ &\qquad + \left(\frac{1}{2}(\bar{k}-\bar{f}) - \dot{\bar{F}}\right) g^5\wedge\left(g^1\wedge g^3 + g^2\wedge g^4\right)\Big]\,. \end{aligned} \tag{3.25}$$

where $F_3^{(0)}$ is the RR 3-form in the KS background. In deriving this result it is convenient to use the representation (2.31) of the 10d metric. In particular the determinant of the 6d metric is given by $g_6 = e^{2m+n}$. Similarly, from the NSNS 3-form equation we have:

$$\mathbf{d}\star_6 Z_3 = 0\,. \tag{3.26}$$

Comparing (3.25) with the l.h.s. of (3.22) we might conclude that the r.h.s. of (3.22) yields expressions for the components of the closed (and co-closed) form Z_3. Notice that having $Z_3 \neq 0$ necessary means that the complex form $G_3 = F_3 + \frac{i}{g_s}H_3$ is not imaginary self dual and therefore the supersymmetry is broken [37, 38]. The most general solution of (3.24) and (3.26) has 3 integration constants and it appears in (2.21). On viewing (2.21) we may conclude that the 3-form on the r.h.s. of (3.22) corresponds to the divergent $(0,3)$-form we have mentioned in the discussion following (2.21). Remarkably, this is the only solution for Z_3, which is consistent with $\dot{\Phi} = 0$. This is evident from the linearized version of the dilaton equation of motion. For $\dot{\Phi} = 0$ it reads:

$$\begin{aligned} F_3^{(0)}\wedge Z_3 = \frac{2X}{2P}\mathbf{M}\Big[&(-\cosh(\tau)+1)(1-F_0(\tau)) + (\cosh(\tau)+1)F_0(\tau) + 2\sinh(\tau)F_0'(\tau)\Big]\times \\ &\times d\tau\wedge g^1\wedge\cdots\wedge g^5 = 0\,, \end{aligned} \tag{3.27}$$

as can be verified by using an explicit expression for $F_0(\tau)$. To find the solution for $\bar{F}(\tau)$, $\bar{f}(\tau)$ and $\bar{k}(\tau)$ note that we already know the solution of the homogeneous part of (3.22). One can read these solutions from (2.21). Let us consider the solution of the inhomogeneous equations in the form:

$$\begin{aligned} \bar{f}(\tau) &= \lambda_1(\tau)f_1(\tau) + \lambda_2(\tau)f_2(\tau) + \lambda_3(\tau)f_3(\tau) \\ \bar{k}(\tau) &= \lambda_1(\tau)k_1(\tau) + \lambda_2(\tau)k_2(\tau) + \lambda_3(\tau)k_3(\tau) \\ \bar{F}(\tau) &= \lambda_1(\tau)F_1(\tau) + \lambda_2(\tau)F_2(\tau) + \lambda_3(\tau)F_3(\tau)\,, \end{aligned} \tag{3.28}$$

where $F_i(\tau)$, $f_i(\tau)$ and $k_i(\tau)$ (for $i = 1,2,3$) appear in (2.21) multiplied by C_i, for example, $F_2(\tau) = (\sinh\tau)^{-1}$ and $F_3(\tau) = 0$. Plugging this into (3.22) we obtain a set of linear equation for $\dot\lambda(\tau)_i$'s. Solving it we get a solution for $\bar F(\tau)$, $\bar f(\tau)$ and $\bar k(\tau)$. The final expressions, which are quite complicated appear in appendix B. Instead we will give the asymptotic solutions at $\tau \to \infty$ and $\tau \to 0$. In the UV we have:

$$
\begin{aligned}
\bar F(\tau) &\approx \mu\left(\frac{3}{4}\tau - 3\right)e^{-\tau/3} + \left(\frac{3}{2}V + V'\right)e^{-\tau} + O(e^{-4\tau/3}) \\
\bar f(\tau) &\approx -\frac{27}{16}\mu e^{-\tau/3} + \left(\frac{V}{2} + V'\right)e^{-\tau} + O(e^{-4\tau/3}) \\
\bar k(\tau) &\approx \frac{27}{16}\mu e^{-\tau/3} - \left(\frac{V}{2} + V'\right)e^{-\tau} + O(e^{-4\tau/3}) \\
\bar f(\tau) + \bar k(\tau) &\approx \mu\left(-3\tau^2 + \frac{9}{2}\tau + \frac{51}{8}\right)e^{-4\tau/3} + O(e^{-2\tau})\,, \qquad (3.29)
\end{aligned}
$$

where V' is a constant proportional to μ. and in the IR:

$$
\bar k(\tau) \approx -2\gamma\tau + O(\tau^3)\,, \qquad \bar f(\tau) \approx \frac{1}{2}\gamma\tau^3 + O(\tau^5)\,, \qquad \bar F(\tau) \approx \gamma\tau^2 + O(\tau^4)\,, \qquad (3.30)
$$

where

$$
\gamma = -\frac{X}{3P}(4P)^2 2^{2/3}\varepsilon^{-8/3}a_0 = -\frac{2^{1/3}}{18}\mu a_0\,. \qquad (3.31)
$$

Here we used the fact that $h_0(\tau) \approx (4P)^2 2^{2/3}\varepsilon^{-8/3}\left(a_0 - a_1\tau^2 + \ldots\right)$. Finally, we arrive at the last equation for the fields:

$$
\begin{aligned}
-2\dot{\bar Y} + 5\dot{\bar q} &= \sqrt{3}e^{-4p_0-6q_0}\Big(-(4\bar p + 6\bar q)\tilde L_0 + (2P - \tilde F_0)\,\bar{\tilde f} + \tilde F_0\bar{\tilde k} + +(\tilde k_0 - \tilde f_0)\,\bar{\tilde F}\Big) + \\
&\quad + \frac{1}{9}\left(\xi_Y + \xi_q\right)e^{4q_0-4p_0-4Y_0}\,. \qquad (3.32)
\end{aligned}
$$

We have already seen that for $\xi_Y = \xi_q = 0$ the self dual 5-form $\tilde F_5$ is given by (2.2) like in the supersymmetric background. The warp function $h(\tau)$ in this case is given by (2.7). Linearizing (2.7) around the supersymmetric configuration and recalling that $h \sim e^{10q-4Y}$ we may re-derive (3.32) for the special case $\xi_Y = \xi_q = 0$. This equation is easily solved once we use the expression for $\bar Y - \bar q$ (see appendix B for the full solution). We obtain:

$$
\bar q(\tau) \approx -\frac{2}{5}\mu\tau e^{-4\tau/3} + O\left(e^{-4\tau/3}\right) \qquad \text{at } \tau \to \infty \qquad (3.33)
$$

and

$$
\bar q = 3^{2/3}\frac{11}{4050}\mu\tau^2 + O(\tau^4) \qquad \text{at } \tau \to 0\,, \qquad (3.34)
$$

This completes our solution for various fields in the non-supersymmetric background. The deformation is controlled by the single parameter μ and all the fields have a regular behavior in the UV and in the IR. There are two non-normalizable modes. The first one is $y(\tau)$ and it is related to the deformation of the 6d metric. The second one is ξ_2 and it is associated with the 3-forms. In the UV we have:

$$
\xi_2 \sim -F + \frac{k-f}{2} \approx -\frac{3}{4}\mu\left(\tau - \frac{25}{4}\right)e^{-\tau/3}\,. \qquad (3.35)
$$

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Both $y(\tau)$ and ξ_2 have dimension $\Delta = 3$ which matches perfectly with the asymptotic behavior of the fields. In the dual gauge theory these operators are dual to the gaugino bilinears. The deformation also involves other fields like $s = f + k$ with a normalizable behavior at $\tau \to \infty$. For example, $s \approx e^{-4\tau/3}$ as expected for an operator with $\Delta = 4$.

Notice also that the field $\bar{Y}$ does not vanish at $\tau = 0$ (namely, $\bar{Y} = C_Y + O(\tau^2)$). This is in the contrast to the IR solution of [13]. We will return to this point in section 6.

4. Vacuum energy

To calculate the vacuum energy of the deformed non-supersymmetric theory we will use the standard AdS/CFT technique [4]. The supergravity dual of the gauge theory hamiltonian is a G_{00} component of the 10d metric. The vacuum energy, therefore can be found by variation of the type-IIB SUGRA action (see appendix A) with respect to G_{00}. This variation vanishes on-shell, except a boundary term. Looking at the supergravity action it is clear that the only such a boundary term will appear from the curvature part of the action. Since the vacuum energy does not depend on the world-volume coordinates we might consider the metric variation in the form:

$$G_{00} \to q G_{00} \,. \tag{4.1}$$

Under this variation the Christoffel connection symbols transform as:

$$\delta\Gamma^{\tau}_{00} = -\frac{1}{2} q G^{\tau\tau} \partial_\tau G_{00} \quad \text{and} \quad \delta\Gamma^{0}_{0\tau} = q G^{00} \partial_\tau G_{00} \tag{4.2}$$

and:[5]

$$\delta\left(\sqrt{-G}\mathcal{R}\right) = \cdots - q\partial_\tau \left[\frac{1}{2}\sqrt{-G} G^{\tau\tau} G^{00} \partial_\tau G_{00}\right], \tag{4.4}$$

where $(\ldots)$ denotes other non-boundary terms which are canceled on-shell by terms coming from the forms part of the action. Note that unlike [11] in our case there is no additional boundary term since the dilaton is taken to be constant. Substituting the on-shell values of the 10d metric ($G_{00} = -h^{-1/2}(\tau)$, $G_{\tau\tau} \sim h^{1/2}(\tau) e^{m(\tau) - \frac{1}{2}n(\tau)}$ and $\sqrt{-G} \sim h^{1/2}(\tau) e^{m(\tau) + \frac{1}{2}n(\tau)}$) we obtain an expression for the vacuum energy:

$$E \sim \lim_{\tau\to\infty} \left(e^{n(\tau)} \partial_\tau \ln h(\tau)\right). \tag{4.5}$$

The divergent result we have found is expected to be canceled out when we compare the vacuum energies of our solution and of the KS background, which we take as a reference. Using that $h \to h_0 + \bar{h}$ and $n \to n_0 + \bar{n}$ we get:

$$\Delta E \sim \left[e^{n_0} \left(\partial_\tau \left(\frac{\bar{h}}{h_0}\right) + \bar{n}\partial_\tau \left(\ln h_0\right)\right)\right]_{\tau\to\infty}. \tag{4.6}$$

[5] here we use the formula:

$$\delta\left(\sqrt{-G}\mathcal{R}\right) = G^{\mu\kappa} G^{\lambda\nu} \delta G_{\kappa\lambda} R_{\mu\nu} \sqrt{-G} + \frac{1}{2}\sqrt{-G}\mathcal{R} G^{\mu\nu} \delta G_{\mu\nu} + \partial_\kappa \left(G^{\mu\nu} \delta\Gamma^{\kappa}_{\mu\nu} \sqrt{-G}\right) - \partial_\nu \left(G^{\mu\nu} \delta\Gamma^{\kappa}_{\kappa\mu} \sqrt{-G}\right). \tag{4.3}$$

The connection between $\bar{n}$ and $\bar{h}$ and the fields we have found in the previous section is:

$$\bar{n} = -4\bar{q} + 4\bar{p} + 4\bar{Y} \qquad \text{and} \qquad \frac{\bar{h}}{h_0} = 10\bar{q} - 4\bar{Y}\,. \tag{4.7}$$

Therefore

$$\Delta E \sim \left[e^{n_0} \left(\left(10\dot{\bar{q}} - 4\dot{\bar{Y}} \right) - \frac{4}{3} \left(-4\bar{q} + 4\bar{p} + 4\bar{Y} \right) \right) \right]_{\tau \to \infty} \sim \mu\,, \tag{4.8}$$

where we used the asymptotic solutions for the fields at $\tau \to \infty$ from the previous section. In (4.8) the term $e^{n_0(\tau)}$ diverges at $\tau \to \infty$ as $e^{4\tau/3}$. This is suppressed by the $e^{-4\tau/3}$ term in the large τ expansion of the fields appearing in the parenthesis which multiply the $e^{n_0(\tau)}$ term. Furthermore, the term linear at τ cancels and we end up with a constant proportional to μ.

5. Dual gauge theory

As was announced in the introduction the deformation of the supergravity background corresponds in the gauge theory to an insertion of the soft supersymmetry breaking gaugino mass terms. The most general gaugino bilinear term has the form of $\mu_+\mathcal{O}_+ + \mu_-\mathcal{O}_- + c.c$ where $\mathcal{O}_\pm \sim Tr[W_{(1)}^2 \pm W_{(2)}^2]$ and $W_{(i)}$, $i = 1, 2$ relate to the $\mathrm{SU}(N+M)$ and $\mathrm{SU}(N)$ gauge groups respectively. Namely, the general deformation is characterized by two complex masses. Our non-supersymmetric deformation of the KS solution derived above is a special case that depends on only one real parameter μ. Since the supergravity identification of the operators $\mathcal{O}_\pm$ is known up to some constants of proportionality we can not determine the precise form of the soft symmetry breaking term.

In the non-deformed supersymmetric theory the $\mathrm{U}(1)_R$ symmetry is broken [39, 40] first by instantons to $\mathbf{Z}_{2M}$ and then further spontaneously broken down to $\mathbf{Z}_2$ by a VEV of the gaugino bilinear. Let us discuss first the latter breaking. We have already seen that on the SUGRA side this fact is manifest from the UV behavior of the complex 3-form $G_3 = F_3 + \frac{i}{g_s}H_3$. The sub-leading term in the expansion of G_3 preserves only the $\mathbf{Z}_2$ part of the $\mathrm{U}(1)_R$ symmetry and it vanishes at infinity like $e^{-\tau}$ matching the expectation from the scalar operator $\mathrm{Tr}(\lambda\lambda)$ of dimension 3 with a non-zero VEV [26]. Plugging the non-supersymmetric solution into G_3 we find that the leading term breaking the $\mathrm{U}(1)_R$ symmetry behaves like $\Delta G_3 = g(\tau)e^{-\tau/3}$, where $g(\tau)$ is some polynomial in τ. This is exactly what one would predict for an operator with $\Delta = 3$ and a non-trivial mass. The second combination of the gaugino bilinears is encoded in the 6d part of the metric. For the 6d metric in (2.31) to preserve the $\mathrm{U}(1)_R$ one has to set $y = 0$. In the supersymmetric deformed conifold metric $y(\tau) = -2e^{-\tau} + \ldots$ similarly to the behavior of the 3-form. In the non-supersymmetric solution $y(\tau)$ goes like $e^{-\tau/3}$ elucidating again that the gaugino combination gets a mass term. Notice also that the non-zero VEVs of the gaugino bilinears are modified by the SUSY breaking deformation. This is evident, for example, from the $Ve^{-\tau}$ term in the UV expansion of $\bar{y}(\tau)$ in (3.12). Clearly, for $V \neq 0$ we have a correction to the VEV in the supersymmetric theory which was encoded in the expansion of $y_0(\tau)$. Similar $e^{-\tau}$ term appears also in the expansion of $\xi_2(\tau)$ and therefore the VEV of the second combination of the gauginos gets modified too.

The spontaneous breaking of the $\mathbf{Z}_{2M}$ discrete group down to the $\mathbf{Z}_2$ subgroup by gaugino condensation results in an $\mathbf{M}$-fold degenerate vacua. This degeneracy is generally lifted by soft breaking mass terms in the action. For small enough masses one can treat the supersymmetry breaking as a perturbation yielding (for a single gauge group) the well-known result [41] that the difference in energy between a non-supersymmetric solution and its supersymmetric reference is given by:

$$\Delta E \sim \mathrm{Re}(\mu C)\,, \tag{5.1}$$

where μ and C are the mass and the gaugino condensate respectively. For the gauge theory dual of the deformed KS solution the vacuum energy will in general be proportional to $\mathrm{Re}(a_+\mu_+C_+ + a_-\mu_-C_-)$ where $C_\pm$ are the expectation values of $\mathcal{O}_\pm$ and $a_\pm$ are some proportionality constants. In the special deformation we are discussing in this paper this reduces to $\mu\mathrm{Re}(a_+C_+ + a_-C_-)$. In the previous section we have derived a result using the SUGRA dual of the gauge theory which has this structure. For the softly broken MN background similar calculations were performed by [11]. In their case the explicit linear dependence on the condensate was demonstrated.

One of the properties of the supersymmetric gauge theory is the space-time independence of the correlation function of two gaugino bilinears. This appears from the supergravity dual description as follows [26]. Consider a perturbation of the complex 2-form $C_2 = C_2^{RR} + iB_2^{NS}$:

$$C_2 \to C_2 + y(x,\tau)\omega_2 \qquad \text{and} \qquad G_3 \to G_3 + y(x,\tau)\wedge\omega_3 + \mathbf{d}y(x,\tau)\wedge\omega_2\,, \tag{5.2}$$

where $\omega_{2,3}$ are given by (2.27) and (2.26) and $y(x,\tau)$ has non-vanishing boundary values. Plugging this forms into the relevant part of the type-IIB action:

$$\int dx d\tau\sqrt{g}\left[G_3G_3^\star + \left(F_5 - \frac{1}{2i}(C_2\wedge G_3^\star - C_2^\star\wedge G_3)\right)\right] \tag{5.3}$$

and integration over τ will *not* lead to a kinematic term $dy(x_1)dy(x_2)$ and therefore the corresponding correlation function will be space-time independent. This derivation is only schematic since there is a mixing between the 3-form modes and the modes coming from metric as we have seen in section 3. Notice, however that this simplified calculation will yield the kinetic term for the deformed non-supersymmetric background, since the complex 3-form is not imaginary self dual in this case. Thus in the non-supersymmetric theory the correlation function will be time-space dependent as one would expect.

6. The plane wave limit

In this section we will construct a Penrose limit of the non-supersymmetric background. Following [15] we will expand the metric around a null geodesic that goes along an equator of the S^3 at $\tau = 0$. The parameter ε appearing in the 6d metric of the deformed conifold and the gauge group parameter $\mathbf{M}$ are both taken to infinity in the PL limit, while keeping finite the mass of the glue-ball:

$$M_{gb} \sim \frac{\varepsilon^{2/3}}{g_s\mathbf{M}\alpha'}\,. \tag{6.1}$$

Let us start with the following general ansatz of a 10d metric:

$$ds^2 = D^{-1}(\tau)\left(dt^2 + dx_i^2\right) + D(\tau)\left(A(\tau)\left(d\tau^2 + g_5^2\right) + B(\tau)\left(g_3^2 + g_4^2\right) + C(\tau)\left(g_1^2 + g_2^2\right)\right), \tag{6.2}$$

where

$$\begin{aligned} A(\tau) &= A_0 + A_1\tau^2 + \cdots \qquad & B(\tau) &= 2A_0 + B_1\tau^2 + \cdots \\ C(\tau) &= \frac{1}{2}A_0\tau^2 + \cdots \qquad & D(\tau) &= D_0 + D_1\tau^2 + \cdots . \end{aligned} \tag{6.3}$$

It can be easily verified that the 10d metric in the KS solution and its non-supersymmetric deformation have this form near $\tau = 0$. Since we expand the metric around the equator of the S^3 it will be useful to switch to a basis of one forms ω_1, ω_2, ω_3 and two additional angles θ and ϕ, related to the 1 forms g_i's by [15]:

$$\begin{aligned} g_5 &= \sin\theta\cos\phi\omega_1 - \sin\theta\sin\phi\omega_2 + \cos\theta\omega_3 \\ \cos\left(\frac{\psi}{2}\right)g_1 + \sin\left(\frac{\psi}{2}\right)g_2 &= \frac{1}{\sqrt{2}}\left(\cos\theta\cos\phi\omega_1 - \cos\theta\sin\phi\omega_2 - \sin\theta\omega_3 - 2\sin\theta d\phi\right) \\ -\sin\left(\frac{\psi}{2}\right)g_1 + \cos\left(\frac{\psi}{2}\right)g_2 &= -\frac{1}{\sqrt{2}}\left(\sin\phi\omega_1 + \cos\phi\omega_2 - 2d\theta\right) \\ \cos\left(\frac{\psi}{2}\right)g_3 + \sin\left(\frac{\psi}{2}\right)g_4 &= \frac{1}{\sqrt{2}}\left(\cos\theta\cos\phi\omega_1 - \cos\theta\sin\phi\omega_2 - \sin\theta\omega_3\right) \\ -\sin\left(\frac{\psi}{2}\right)g_3 + \cos\left(\frac{\psi}{2}\right)g_4 &= -\frac{1}{\sqrt{2}}\left(\sin\phi\omega_1 + \cos\phi\omega_2\right). \end{aligned} \tag{6.4}$$

In terms of the S^3 angle coordinates (θ', ϕ', ψ') the geodesic lies at $\theta' = 0$ and is generated by $\phi_+ = \frac{1}{2}(\phi' + \psi')$. Under re-scaling $\theta' \to \theta'/L$ the 1-forms ω_1 and ω_2 will go like $1/L$ and for ω_3 we obtain:

$$\omega_3 = 2d\phi_+ - \frac{1}{2}\left(\frac{\theta'}{L}\right)^2 d\phi' . \tag{6.5}$$

In order to take the Penrose limit we define new coordinates:

$$\begin{aligned} u &= (D_0A_0)^{1/2}\,\tau\sin\theta e^{i(\phi+\phi_+)} \qquad & z &= (D_0A_0)^{1/2}\,\tau\cos\theta \\ v &= (D_0A_0)^{1/2}\,\theta' e^{i(\phi'-\phi_+)} \qquad & x_i &\to \tfrac{x_i}{D_0^{1/2}} \end{aligned} \tag{6.6}$$

together with

$$t = x_+ \quad \text{and} \quad \phi_+ = \frac{1}{2D_0A_0^{1/2}}\left(x_+ - 2D_0x_-\right). \tag{6.7}$$

Finally, re-scaling $D_0 \to D_0L^2$ and $A_0 \to A_0L^{-4}$ and taking $L \to \infty$ we arrive at the following pp-wave metric:

$$\begin{aligned} ds^2 &= -4dx_-dx_+ - m_0^2\left(v\bar{v} + \left(-\frac{4A_1}{A_0} - \frac{8D_1}{D_0}\right)z^2 + \left(-\frac{2B_1}{A_0} - \frac{8D_1}{D_0}\right)u\bar{u}\right)dx_+^2 + \\ &\quad + dx_i^2 + dz^2 + dud\bar{u} + dvd\bar{v}, \end{aligned} \tag{6.8}$$

where $m_0 = (2D_0A_0^{1/2})^{-1}$ remains finite in the $L \to \infty$ limit. Now we are in a position to take the PL of the non-supersymmetric background we have found in section 3. Note that:

$$D(\tau) = h^{1/2}(\tau), \qquad A(\tau) = e^{m(\tau) - \frac{1}{2}n(\tau)}, \qquad B(\tau) = e^{\frac{1}{2}n(\tau) - y(\tau)}, \qquad C(\tau) = e^{\frac{1}{2}n(\tau) + y(\tau)}, \tag{6.9}$$

and the relations between $h(\tau)$,$m(\tau)$, $n(\tau)$ and the fields $Y(\tau)$, $p(\tau)$ and $q(\tau)$ are given in (2.33). The final result is:

$$\begin{aligned} ds^2 &= -4dx_-dx_+ + dx_i^2 + dz^2 + dud\bar{u} + dvd\bar{v} + \\ &\quad -m_0^2\left[v\bar{v} + \left(\left(\frac{4a_1}{a_0}-\frac{4}{5}\right) - 8\frac{3^{2/3}}{135}\mu\right)z^2 + \left(\left(\frac{4a_1}{a_0}-\frac{3}{5}\right) + 4\frac{3^{2/3}}{135}\mu\right)u\bar{u}\right]dx_+^2\,, \end{aligned} \tag{6.10}$$

where

$$m_0^2 = \frac{3^{1/3}\varepsilon^{4/3}}{2(g_s\mathbf{M}\alpha')^2 a_0}\,(1+2C_Y)\,. \tag{6.11}$$

Recall that C_Y is a numerical constant proportional to μ. As expected for $\mu = 0$ we recover the result of the supersymmetric case [15]. We see that all the world-sheet masses (m_v,m_z and m_u) depend on the supersymmetry breaking parameter. Under the Penrose limit the 3-forms read:

$$\begin{aligned} F_3 &= \frac{3im_0}{\sqrt{2}g_s}\left(\frac{a_1}{a_0}\right)^{1/2} dx_+ \wedge \left(\left(\frac{1}{3}+4\gamma)du\wedge d\bar{u} + dv\wedge d\bar{v}\right)\right) \\ H_3 &= \frac{im_0}{\sqrt{2}}\left(\frac{a_1}{a_0}\right)^{1/2} dx_+ \wedge (1-6\gamma)\,(du\wedge d\bar{v} - d\bar{u}\wedge dv) \end{aligned} \tag{6.12}$$

and the complex 3-form is given by:

$$\begin{aligned} G_3{=}F_3{+}\frac{i}{g_s}H_3 &= \frac{im_0}{\sqrt{2}g_s}\left(\frac{a_1}{a_0}\right)^{1/2} \times \\ &\quad \times dx_+ \wedge \Big[((1+12\gamma)du\wedge d\bar{u} + dv\wedge d\bar{v}) + i(1-6\gamma)(du\wedge d\bar{v} - d\bar{u}\wedge dv)\Big]\,, \end{aligned} \tag{6.13}$$

where $\gamma = -\frac{1}{18}2^{1/3}\mu a_0$.

As a non-trivial check of our solution we can verify that the equation of motion:

$$R_{++} = \frac{g_s^2}{4}\,(G_3)_{+ij}\,(\bar{G}_3)_+^{\;ij} \tag{6.14}$$

is satisfied. Indeed:

$$R_{++} = m_0^2\left[2 + \left(\left(\frac{4a_1}{a_0}-\frac{4}{5}\right) - 8\frac{3^{2/3}}{135}\mu\right) + 2\left(\left(\frac{4a_1}{a_0}-\frac{3}{5}\right) + 4\frac{3^{2/3}}{135}\mu\right)\right] = 12m_0^2\,\frac{a_1}{a_0} \tag{6.15}$$

and the norm of the 3-form is:

$$(G_3)_{+ij}\,(\bar{G}_3)_+^{\;ij} = m_0^2\,\frac{a_1}{a_0}\,4\left((1+12\gamma)^2 + 9 + 2(1-6\gamma)^2\right) = 48m_0^2\,\frac{a_1}{a_0} + O(\gamma^2) \tag{6.16}$$

matching perfectly with the R_{++}. Notice that in the last equation we have neglected corrections of the 2nd order in the deformation.

7. The plane wave string theory and the annulons

The string theory associated with the plane wave background described in the previous section is quite similar to that associated with the PL limit of the KS background. The bosonic sector includes three massless fields that correspond to the spatial directions on the world-volume of the D3 branes. Their masslessness is attributed to the translational invariance of the original metric and the fact that the null geodesic is at constant τ. The rest five coordinates are massive. Altogether the bosonic spectrum takes the form

$$\begin{aligned} \omega_n^i &= n \qquad \text{for } i=1,2,3\,; \qquad \omega_n^z = \sqrt{n^2+\hat{m}_z^2}\,; \\ \omega_n^{u,v} &= \sqrt{n^2+\frac{1}{2}\left(\hat{m}_v^2+\hat{m}_u^2\right) \pm \sqrt{\frac{1}{4}\left(\hat{m}_v^2-\hat{m}_u^2\right)^2+n^2\hat{m}_B^2}}\,; \end{aligned} \tag{7.1}$$

where

$$\begin{aligned} \hat{m}_v &= p^+\alpha' m_0\,, \qquad \hat{m}_B = \sqrt{2}p^+\alpha' m_0\left(\frac{a_1}{a_0}\right)^{1/2}(1-6\gamma)\,, \\ \hat{m}_z &= p^+\alpha' m_0\sqrt{\frac{4a_1}{a_0}-\frac{4}{5}-8\frac{3^{2/3}}{135}\mu} \qquad \text{and} \qquad \hat{m}_u = p^+\alpha' m_0\sqrt{\frac{4a_1}{a_0}-\frac{3}{5}+4\frac{3^{2/3}}{135}\mu}\,. \end{aligned} \tag{7.2}$$

The difference between the bosonic spectrum of the deformed model and that of [15] is the shift of the masses of the $z, v, \bar{v}, u, \bar{u}$ fields. The sum of the $mass^2$ of the individual fields $\sum m^2 = 12m_0^2\frac{a_1}{a_0}$ has the same form as the sum in the supersymmetric case apart from the modification of m_0 (6.11). The modification of m_0 is also responsible for the deviation of the deformed string tension with from the supersymmetric one since the string tension $T_s \sim g_s\mathbf{M}m_0^2$.

The fermionic spectrum takes the form

$$\begin{aligned} \omega_n^k &= \sqrt{n^2+\hat{m}_B^2\left(\frac{1+3\gamma}{1-6\gamma}\right)^2} \approx \sqrt{n^2+\hat{m}_B^2(1+18\gamma)} \qquad \text{for } k=1,\ldots,4\,; \\ \omega_n^l &= \sqrt{n^2+\frac{1}{4}\hat{m}_B^2} \pm \frac{1}{2}\hat{m}_B \qquad \text{for } l=1,2\,. \end{aligned} \tag{7.3}$$

Comparing the bosonic and fermionic masses we observe that like in the undeformed KS model there is no linearly realized world-sheet supersymmetry and the hence there is a non-vanishing zero point energy. However, up to deviations linear in μ the sum of the square of the frequencies of the bosonic and fermionic modes match. Since this property follows in fact from the relation between R_{++} and $(G_3)_{+ij}\left(\bar{G}_3\right)_+^{ij}$ it should be a universal property of any plane wave background.

Surprisingly we find that the fermionic spectrum admits two fermionic zero modes $\omega_0^{l=1,2}$ exactly like in the supersymmetric case. The fermionic zero modes in the spectrum of the latter case were predicted [15] upon observing that the hamiltonian still commutes with the four supercharges that correspond to the four dimensional $\mathcal{N}=1$ supersymmetric gauge theory. This implies that four supersymmetries out of the sixteen supersymmetries of plane wave solution commute with the hamiltonian giving rise to the four zero-frequency modes and a four dimensional Hilbert sub-space of (two bosonic and two fermionic) degenerate states. One might have expected that in the PL of the deformed theory the fermionic zero modes will be lifted by an amount proportional to the supersymmetry breaking parameter. Our results, however, contradict this expectation. In the dual field theory this implies that even though the full theory is non-supersymmetric, the sector of states with large J charge admits supersymmetry. As will be discussed below these states are characterized by their large mass which is proportional to J. Presumably, in this limit of states of large mass the impact of the small gaugino mass deformations is washed away. For instance one can estimate that the ratio of the boson fermion mass difference to the mass of the annulon scales like μ/J and since μ has to be small and $J \to \infty$ this ratio is negligible.

Note that the fermionic zero modes are in accordance with the the criteria presented in [22]. However, the metric and the 3-form given in [22] do not coincide with our results, because of the factor of C_Y in the expression for m_0^2.

Since apart from the modifications of the fermionic and bosonic frequencies the string hamiltonian we find has the same structure as the one found for the KS case, the analysis of the corresponding gauge theory states also follows that of [15]. We will not repeat here this analysis, but rather just summarize its outcome:

- The ground state of the string corresponds to the annulon. This hadron which carries a large J charge is also very massive since its mass is given by

$$M_{\text{annulon}} = m_0 J \tag{7.4}$$

  Obviously, the only difference between the annulon of the deformed theory in comparison with the supersymmetric one is the modification of m_0.

- The annulon can be described as a ring composed of J constituents each having a mass (in the mean field of all the others) of m_0.

- The annulon which is a space-time scalar has a fermionic superpartner of the same mass. The same holds for the rest of the bosonic states.

- The string hamiltonian has a term $P_i^2/2m_0J$ that describes a non-relativistic motion of the annulons.

- The annulons admit stringy ripples. The spacing between these excitations are proportional to T_s/M_{annulon}.

- The string hamiltonian describes also excitations that correspond to the addition of small number of different constituents on top of the J basic ones.

8. Discussion

In this paper we have found an explicit solution for the first order deformation of the KS supergravity background. This deformation breaks the supersymmetry as one can see, for example, from the structure of the deformed complex 3-form, which is not imaginary self dual as required by the type-IIB BPS equations. We have verified that the solution is

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regular in the IR and the UV and the leading order UV behavior of various fields matches their conformal dimensions. We have also identified two fields with a non-normalizable modes. In the dual gauge theory these fields correspond to the gaugino bilinears and therefore the deformation is related to the insertion of the softly supersymmetry breaking gaugino mass terms. Theses masses remove the degeneracy of the vacuum. Using the dual supergravity description we have checked that the lifting of the vacuum energy satisfies the prediction for $\mathcal{N}=1$ theories. Finally, we have investigated the plane-wave limit of the non-supersymmetric background finding that there are two fermionic zero frequencies exactly like in the PL of the supersymmetric solution.

There are plenty of open questions that deserve further investigation. Let us mention only few of them. The solution we have found is by no means the most general one. It is characterized by one real gaugino mass whereas in general one can introduce two complex masses. It will be interesting to determine the corresponding general solutions of the equations of motion. In the laboratory of the non-supersymmetric solution we have found, it will be interesting to "measure" certain properties of the gauge dynamics like Wilson loops, 't Hooft loops, baryonic configurations, fundamental quarks via D brane probes etc. Another interesting question to explore is whether, the surprising supersymmetry of the gauge sector dual of the annulons in the overall non-supersymmetric theory, will survive in the presence of $1/J$corrections.

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A. Type-IIB equations of motion

In Einstein frame the bosonic part of the type-IIB action is:

$$\frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\mathcal{R}-\frac{1}{4\kappa^2}\int d^{10}x\Big[\mathbf{d}\Phi\wedge\star\mathbf{d}\Phi+e^{2\Phi}\mathbf{d}C_0\wedge\star\mathbf{d}C_0+g_s e^{-\Phi}H_3\wedge\star H_3+ \\ +g_s e^{\Phi}\tilde{F}_3\wedge\star\tilde{F}_3+\frac{g_s^2}{2}\tilde{F}_5\wedge\star\tilde{F}_5+g_s^2 C_4\wedge H_3\wedge F_3\Big]. \quad (A.1)$$

The field equations are

$$\begin{aligned}
\mathbf{d}\star\mathbf{d}\Phi &= e^{2\Phi}\mathbf{d}C_0\wedge\star\mathbf{d}C_0-\frac{1}{2}g_s e^{-\Phi}H_3\wedge\star H_3+\frac{1}{2}g_s e^{\Phi}\tilde{F}_3\wedge\star\tilde{F}_3,\\
\mathbf{d}\left(e^{2\Phi}\star\mathbf{d}C_0\right) &= -g_s e^{\Phi}H_3\wedge\star\tilde{F}_3,\\
\mathbf{d}\star\left(e^{\Phi}\tilde{F}_3\right) &= g_s F_5\wedge H_3,\\
\mathbf{d}\star\left(e^{-\Phi}H_3-C_0 e^{\Phi}\tilde{F}_3\right) &= -g_s F_5\wedge F_3,
\end{aligned}$$

$$\begin{aligned}
\mathbf{d}\star\tilde{F}_5 &= -F_3\wedge H_3,\\
R_{mn} &= \frac{1}{2}\partial_m\Phi\partial_n\Phi+\frac{1}{2}e^{2\Phi}\partial_m C_0\partial_n C_0+\frac{g_s^2}{96}\tilde{F}_{mpqrs}\tilde{F}_m{}^{pqrs}+\\
&\quad+\frac{g_s}{4}\left(e^{-\Phi}H_{mpq}H_m{}^{pq}+e^{\Phi}\tilde{F}_{mpq}\tilde{F}_m{}^{pq}-\right)\\
&\quad-\frac{g_s}{48}g_{mn}\left(e^{-\Phi}H_{mpq}H^{mpq}+e^{\Phi}\tilde{F}_{mpq}\tilde{F}^{mpq}\right).
\end{aligned}\tag{A.2}$$

Here

$$\begin{aligned}
\tilde{F}_3 &= F_3-C_0H_3\,, \qquad F_3=\mathbf{d}C_2\,,\\
\tilde{F}_5 &= F_5-C_2\wedge H_3\,, \qquad F_5=\mathbf{d}C_4\\
H_3 &= \mathbf{d}B_2\,.
\end{aligned}\tag{A.3}$$

The Bianchi identities are:

$$\begin{aligned}
\mathbf{d}\tilde{F}_3 &= -\mathbf{d}C_0\wedge H_3\,,\\
\mathbf{d}\tilde{F}_5 &= -F_3\wedge H_3
\end{aligned}\tag{A.4}$$

and the 5-form is self dual:

$$\star\tilde{F}_5 = \tilde{F}_5\,.\tag{A.5}$$

B. The explicit solutions for $f(\tau)$, $k(\tau)$, $F(\tau)$ and $q(\tau)$

The functions $f(\tau)$, $k(\tau)$, $F(\tau)$ are given by (3.28), where:

$$\begin{aligned}
\lambda_1 &= \frac{1}{2}\int_\tau^\infty\frac{k_0'(x)+f_0'(x)}{\sinh x}\bar{y}(x)dx\\
\lambda_2 &= \frac{1}{2}\int_0^\tau\left(\frac{1}{2}\left(\cosh x-\frac{x}{\sinh x}\right)(k_0'(x)+f_0'(x))\bar{y}(x)-\frac{2X}{2P}h_0(x)\sinh^2 x\right)dx\\
\lambda_3 &= -\int_\tau^\infty\Bigg((f_0'(x)-k_0'(x))\bar{y}(x)-\frac{1}{2}(k_1(x)+f_1(x))\lambda_1'(x)-\\
&\qquad-\frac{1}{2}(k_2(x)+f_2(x))\lambda_2'(x)++\frac{2X}{2P}h_0(x)\Bigg)dx\,.
\end{aligned}\tag{B.1}$$

For $q(\tau)$ we have:

$$\begin{aligned}
\bar{q(\tau)} &= \frac{1}{\gamma(\tau)}\int_0^\tau\gamma(x)\Bigg(\frac{2}{3}(\dot{\bar{Y}}-\dot{\bar{q}})+\frac{\sqrt{3}}{3}e^{-4p_0(x)-6q_0(x)}\tilde{L}_0(x)\times\\
&\qquad\times\Bigg(-4\bar{p}(x)++\frac{1-F_0(x)}{L_0(x)}\bar{f}(x)+\frac{F_0(x)}{L_0(x)}\bar{k}(x)+\\
&\qquad+\frac{k_0(x)-f_0(x)}{L_0(x)}\bar{F}(x)\Bigg)\Bigg)dx
\end{aligned}\tag{B.2}$$

with

$$\gamma(\tau)=e^{2\sqrt{3}\int_{\tau_0}^\tau e^{-4p_0(x)-6q_0(x)}\tilde{L}_0(x)dx}\,.\tag{B.3}$$

C. The complex $(0,3)$ form on the deformed conifold

In this section we demonstrate by an explicit calculation that the complex 3-form $G_3 \equiv F_3 + iH_3$ given by (2.17), (2.18) and (2.21) for $C_1 = -\frac{1}{2}$ and $C_2 = C_3 = 0$ is a $(0,3)$ form on the deformed conifold space. In deriving this result we will use the complex structure of the deformed conifold identified in [24] (one might alternatively use the results of [25] or [23]). To re-write the Kähler form and the metric in the standard form one starts by introducing left-invariant one forms $\{h_i, \tilde{h}_i\}$ for $(i = 1,2,3)$ on the group $\mathrm{SU}(2) \times \mathrm{SU}(2)$. In terms of the 1-forms g_i's we have [24]:

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} -\cos\frac{\psi}{2} & -\sin\frac{\psi}{2} \\ -\sin\frac{\psi}{2} & \cos\frac{\psi}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}(g_1 + g_3) \\ \frac{1}{\sqrt{2}}(g_2 + g_4) \end{pmatrix}$$
$$\begin{pmatrix} \tilde{h}_1 \\ \tilde{h}_2 \end{pmatrix} = \begin{pmatrix} \cos\frac{\psi}{2} & \sin\frac{\psi}{2} \\ -\sin\frac{\psi}{2} & \cos\frac{\psi}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}}(g_3 - g_1) \\ \frac{1}{\sqrt{2}}(g_4 - g_2) \end{pmatrix} .$$

and

$$h_3 + \tilde{h}_3 = g_5 \, . \tag{C.1}$$

Then the Kähler 2-form Ω and the metric will be given by:

$$\Omega = E_1 \wedge E_2 + E_3 \wedge E_4 + E_5 \wedge E_6 \qquad \text{and} \qquad ds_6^2 = E_1^2 + E_2^2 + E_3^2 + E_4^2 + E_5^2 + E_6^2 \, , \tag{C.2}$$

where

$$\begin{aligned} E_1 &= A(\tau)\left(\alpha(\tau)h_1 - \beta(\tau)\tilde{h}_1\right) & E_2 &= A(\tau)\left(\alpha(\tau)h_2 + \beta(\tau)\tilde{h}_2\right) \\ E_3 &= A(\tau)\left(-\beta(\tau)h_1 + \alpha(\tau)\tilde{h}_1\right) & E_2 &= A(\tau)\left(\beta(\tau)h_2 + \alpha(\tau)\tilde{h}_2\right) \\ E_5 &= B(\tau)d\tau & E_6 &= B(\tau)(h_3 + \tilde{h}_3) \end{aligned} \tag{C.3}$$

with

$$\begin{aligned} A(\tau) &= \frac{1}{2}\epsilon^{2/3}\left(\coth(\tau)(\sinh(2\tau) - 2\tau)^{1/3}\right)^{1/2} \\ B(\tau) &= \frac{1}{\sqrt{3}}\epsilon^{2/3}\sinh(\tau)(\sinh(2\tau) - 2\tau)^{-1/3} \\ \alpha(\tau) &= \left(\frac{1}{2}\left(1 + \tanh(\tau)\right)\right)^{1/2} \qquad \text{and} \qquad \beta(\tau) = \left(\frac{1}{2}\left(1 - \tanh(\tau)\right)\right)^{1/2} . \end{aligned} \tag{C.4}$$

The complex structure is defined in terms of the 1-forms E_i's as follows:

$$\begin{aligned} J(E_1) &= E_2 \, , \quad J(E_2) = -E_1 \, , \\ J(E_3) &= E_4 \, , \quad J(E_4) = -E_3 , \\ J(E_5) &= E_6 \, , \quad J(E_6) = -E_5 \, . \end{aligned} \tag{C.5}$$

The integrability of this structure can be verified by a straightforward computation [24]. We are mainly interested in the complex $(0,3)$ form:

$$\eta_{(0,3)} = \frac{2\sqrt{3}}{\epsilon^2}\dot{\mathbf{M}}(E_1 - iE_2) \wedge (E_3 - iE_4) \wedge (E_5 - iE_6) \, . \tag{C.6}$$

On plugging the expressions for E_i's a somewhat lengthy calculation leads to:

$$\begin{aligned}
\mathrm{Re}(\eta_{(0,3)}) &= \frac{2\sqrt{3}}{\epsilon^2}\mathbf{M}\left[(E_1\wedge E_3 - E_2\wedge E_4)\wedge E_5 - (E_2\wedge E_3 + E_1\wedge E_4)\wedge E_6\right] \\
&= \frac{2\sqrt{3}}{\epsilon^2}\mathbf{M}A^2(\tau)B(\tau)\Big[-(\alpha^2(\tau)-\beta^2(\tau))d\tau\wedge\left(g^1\wedge g^3 + g^2\wedge g^4\right) + \\
&\qquad + g_5\wedge((-1{+}2\alpha(\tau)\beta(\tau))g_1\wedge g_2 + (1{+}2\alpha(\tau)\beta(\tau))g_3\wedge g_4)\Big] \\
&= \mathbf{M}\Big[-\frac{1}{2}\sinh(\tau)d\tau\wedge\left(g^1\wedge g^3+g^2\wedge g^4\right) + \\
&\qquad + \frac{1}{2}(1-\cosh(\tau))g_5\wedge g_1\wedge g_2 + \frac{1}{2}(1+\cosh(\tau))g_5\wedge g_3\wedge g_4\Big] \qquad \text{(C.7)}
\end{aligned}$$

and

$$\begin{aligned}
\mathrm{Im}(\eta_{(0,3)}) &= \frac{2\sqrt{3}}{\epsilon^2}\mathbf{M}\left[-(E_2\wedge E_3 + E_1\wedge E_4)\wedge E_5 + (-E_1\wedge E_3 + E_2\wedge E_4)\wedge E_6\right] \\
&= \frac{2\sqrt{3}}{\epsilon^2}\mathbf{M}A^2(\tau)B(\tau)\Big[(\alpha^2(\tau)-\beta^2(\tau))g_5\wedge\left(g^1\wedge g^3+g^2\wedge g^4\right) + \\
&\qquad + d\tau\wedge((-1{+}2\alpha(\tau)\beta(\tau))g_1\wedge g_2 {+} (1{+}2\alpha(\tau)\beta(\tau))g_3\wedge g_4)\Big] \\
&= \mathbf{M}\Big[\frac{1}{2}\sinh(\tau)g_5\wedge\left(g^1\wedge g^3+g^2\wedge g^4\right) + \frac{1}{2}(1-\cosh(\tau))d\tau\wedge g_1\wedge g_2 + \\
&\qquad + \frac{1}{2}(1+\cosh(\tau))d\tau\wedge g_3\wedge g_4\Big]. \qquad \text{(C.8)}
\end{aligned}$$

This means that:

$$\begin{aligned}
F(\tau) &= \frac{1}{2}(1-\cosh(\tau)) \\
f(\tau) &= \frac{1}{2}(\sinh(\tau)-\tau) \\
k(\tau) &= \frac{1}{2}(\sinh(\tau)+\tau) \qquad \text{(C.9)}
\end{aligned}$$

which matches (2.21) for $C_1 = -1/2$ and $C_2 = C_3 = 0$.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[5] J.M. Maldacena and C. Núñez, *Towards the large-N limit of pure N = 1 super Yang-Mills*, *Phys. Rev. Lett.* **86** (2001) 588 [hep-th/0008001].

[6] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularities*, *J. High Energy Phys.* **08** (2000) 052 [hep-th/0007191].

[7] M. Bertolini, *Four lectures on the gauge-gravity correspondence*, hep-th/0303160.

[8] F. Bigazzi, A.L. Cotrone, M. Petrini and A. Zaffaroni, *Supergravity duals of supersymmetric four dimensional gauge theories*, *Riv. Nuovo Cim.* **25N12** (2002) 1–70 [hep-th/0303191].

[9] A.H. Chamseddine and M.S. Volkov, *Non-abelian BPS monopoles in $N = 4$ gauged supergravity*, *Phys. Rev. Lett.* **79** (1997) 3343 [hep-th/9707176].

[10] O. Aharony, E. Schreiber and J. Sonnenschein, *Stable non-supersymmetric supergravity solutions from deformations of the Maldacena-Núñez background*, *J. High Energy Phys.* **04** (2002) 011 [hep-th/0201224].

[11] N. Evans, M. Petrini and A. Zaffaroni, *The gravity dual of softly broken $N = 1$ super Yang-Mills*, *J. High Energy Phys.* **06** (2002) 004 [hep-th/0203203].

[12] S.S. Gubser, A.A. Tseytlin and M.S. Volkov, *Non-abelian 4-d black holes, wrapped 5-branes and their dual descriptions*, *J. High Energy Phys.* **09** (2001) 017 [hep-th/0108205].

[13] R. Apreda, *Non supersymmetric regular solutions from wrapped and fractional branes*, hep-th/0301118.

[14] V. Borokhov and S.S. Gubser, *Non-supersymmetric deformations of the dual of a confining gauge theory*, *J. High Energy Phys.* **05** (2003) 034 [hep-th/0206098].

[15] E.G. Gimon, L.A. Pando Zayas, J. Sonnenschein and M.J. Strassler, *A soluble string theory of hadrons*, *J. High Energy Phys.* **05** (2003) 039 [hep-th/0212061].

[16] R. Güven, *Plane wave limits and t-duality*, *Phys. Lett.* **B 482** (2000) 255 [hep-th/0005061].

[17] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A new maximally supersymmetric background of iib superstring theory*, *J. High Energy Phys.* **01** (2002) 047 [hep-th/0110242].

[18] R.R. Metsaev, *Type iib Green-Schwarz superstring in plane wave Ramond-Ramond background*, *Nucl. Phys.* **B 625** (2002) 70 [hep-th/0112044].

[19] D. Berenstein, J.M. Maldacena and H. Nastase, *Strings in flat space and PP waves from $N = 4$ super Yang-Mills*, *J. High Energy Phys.* **04** (2002) 013 [hep-th/0202021].

[20] R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave Ramond-Ramond background*, *Phys. Rev.* **D 65** (2002) 126004 [hep-th/0202109].

[21] M. Blau, J. Figueroa-O'Farrill and G. Papadopoulos, *Penrose limits, supergravity and brane dynamics*, *Class. and Quant. Grav.* **19** (2002) 4753 [hep-th/0202111].

[22] R. Apreda, F. Bigazzi and A.L. Cotrone, *Strings on PP-waves and hadrons in (softly broken) $N = 1$ gauge theories*, *J. High Energy Phys.* **12** (2003) 042 [hep-th/0307055].

[23] C01-07-20.4 collaboration, C.P. Herzog, I.R. Klebanov and P. Ouyang, *Remarks on the warped deformed conifold*, hep-th/0108101.

[24] G. Papadopoulos and A.A. Tseytlin, *Complex geometry of conifolds and 5-brane wrapped on 2- sphere*, *Class. and Quant. Grav.* **18** (2001) 1333 [hep-th/0012034].

[25] M. Cvetič, G.W. Gibbons, H. Lu and C.N. Pope, *Ricci-flat metrics, harmonic forms and brane resolutions*, *Commun. Math. Phys.* **232** (2003) 457 [hep-th/0012011].

[26] A. Loewy and J. Sonnenschein, *On the holographic duals of $N = 1$ gauge dynamics*, *J. High Energy Phys.* **08** (2001) 007 [hep-th/0103163].

[27] I.R. Klebanov and A.A. Tseytlin, *Gravity duals of supersymmetric* $\mathrm{SU}(N) \times \mathrm{SU}(N + M)$ *gauge theories*, *Nucl. Phys.* **B 578** (2000) 123 [hep-th/0002159].

[28] L.A. Pando Zayas and A.A. Tseytlin, *3-branes on resolved conifold*, *J. High Energy Phys.* **11** (2000) 028 [hep-th/0010088].

[29] F. Bigazzi, L. Girardello and A. Zaffaroni, *A note on regular type 0 solutions and confining gauge theories*, *Nucl. Phys.* **B 598** (2001) 530 [hep-th/0011041].

[30] A. Ceresole, G. Dall'Agata, R. D'Auria and S. Ferrara, *Spectrum of type-IIB supergravity on $AdS_5 \times T^{11}$: predictions on $N = 1$ sCFT's*, *Phys. Rev.* **D 61** (2000) 066001 [hep-th/9905226].

[31] S. Ferrara, R. Kallosh and A. Strominger, *$N = 2$ extremal black holes*, *Phys. Rev.* **D 52** (1995) 5412 [hep-th/9508072].

[32] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem*, *Adv. Theor. Math. Phys.* **3** (1999) 363 [hep-th/9904017].

[33] K. Skenderis and P.K. Townsend, *Gravitational stability and renormalization-group flow*, *Phys. Lett.* **B 468** (1999) 46 [hep-th/9909070].

[34] O. DeWolfe, D.Z. Freedman, S.S. Gubser and A. Karch, *Modeling the fifth dimension with scalars and gravity*, *Phys. Rev.* **D 62** (2000) 046008 [hep-th/9909134].

[35] J. de Boer, E. Verlinde and H. Verlinde, *On the holographic renormalization group*, *J. High Energy Phys.* **08** (2000) 003 [hep-th/9912012].

[36] M. Wijnholt and S. Zhukov, *On the uniqueness of black hole attractors*, hep-th/9912002.

[37] M. Grana and J. Polchinski, *Supersymmetric three-form flux perturbations on AdS_5*, *Phys. Rev.* **D 63** (2001) 026001 [hep-th/0009211].

[38] S.S. Gubser, *Supersymmetry and F-theory realization of the deformed conifold with three-form flux*, hep-th/0010010.

[39] I.R. Klebanov, P. Ouyang and E. Witten, *A gravity dual of the chiral anomaly*, *Phys. Rev.* **D 65** (2002) 105007 [hep-th/0202056].

[40] C.P. Herzog, I.R. Klebanov and P. Ouyang, *D-branes on the conifold and $N = 1$ gauge/gravity dualities*, hep-th/0205100.

[41] A. Masiero and G. Veneziano, *Split light composite supermultiplets*, *Nucl. Phys.* **B 249** (1985) 593.